Paper Reference 9MA0/02
Pearson Edexcel
Level 3 GCE

Mathematics

Advanced

PAPER 2: Pure Mathematics 2

Tuesday 13 June 2023 – Afternoon

Time: 2 hours

YOU MUST HAVE:
Mathematical Formulae and Statistical Tables (Green), calculator

YOU WILL BE GIVEN:
Diagram Booklet
Answer Booklet

X72805A

Pearson

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

INSTRUCTIONS

In the boxes on the Answer Booklet and on the Diagram Booklet, write your name, centre number and candidate number.

Answer ALL questions and ensure that your answers to parts of questions are clearly labelled.

Answer the questions in the spaces provided in the Answer Booklet – there may be more space than you need.

Do NOT write on the Question Paper.

You should show sufficient working to make your methods clear.
Answers without working may not gain full credit.

Inexact answers should be given to three significant figures unless otherwise stated.

Turn over

INFORMATION

A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.

There are **15** questions in this Question Paper.
The total mark for this paper is **100**.

The marks for EACH question are shown in brackets – use this as a guide as to how much time to spend on each question.

You may be provided with a model for Question 11
It is NOT accurate.

ADVICE

Read each question carefully before you start to answer it.

Try to answer every question.

Check your answers if you have time at the end.

1. $f(x) = x^3 + 2x^2 - 8x + 5$

(a) Find $f''(x)$

(2 marks)

(b) (i) Solve $f''(x) = 0$

(ii) Hence find the range of values of x for which $f(x)$ is concave.

(2 marks)

(Total for Question 1 is 4 marks)

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2. A sequence $u_1, u_2, u_3 \ldots$ is defined by

$$u_1 = 35$$

$$u_{n+1} = u_n + 7\cos\left(\frac{n\pi}{2}\right) - 5(-1)^n$$

(a) (i) Show that

$$u_2 = 40$$

(ii) Find the value of u_3 and the value of u_4

(3 marks)

Given that the sequence is periodic with order 4

(b) (i) write down the value of u_5

(ii) find the value of

$$\sum_{r=1}^{25} u_r$$

(3 marks)

(Total for Question 2 is 6 marks)

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3. Given that

$$\log_2(x + 3) + \log_2(x + 10) = 2 + 2\log_2 x$$

(a) show that

$$3x^2 - 13x - 30 = 0$$

(3 marks)

(b) (i) Write down the roots of the equation

$$3x^2 - 13x - 30 = 0$$

(ii) Hence state which of the roots in part (b)(i) is not a solution of

$$\log_2(x + 3) + \log_2(x + 10) = 2 + 2\log_2 x$$

giving a reason for your answer.

(2 marks)

(Total for Question 3 is 5 marks)

4. Coffee is poured into a cup.

The temperature of the coffee, H °C, t minutes after being poured into the cup is modelled by the equation

$$H = Ae^{-Bt} + 30$$

where A and B are constants.

Initially, the temperature of the coffee was 85 °C

(a) State the value of A

(1 mark)

Initially, the coffee was cooling at a rate of $7 \cdot 5$ °C per minute.

(b) Find a complete equation linking H and t, giving the value of B to 3 decimal places.

(3 marks)

(Total for Question 4 is 4 marks)

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5. The curve C has equation

$$y = f(x)$$

The curve

- passes through the point $P(3, -10)$
- has a turning point at P

Given that

$$\frac{dy}{dx} = 2x^3 - 9x^2 + 5x + k$$

where k is a constant,

(a) show that $k = 12$

(2 marks)

(b) Hence find the coordinates of the point where C crosses the y–axis.

(3 marks)

(Total for Question 5 is 5 marks)

6. Relative to a fixed origin O,

- A is the point with position vector $12\underline{i}$
- B is the point with position vector $16\underline{j}$
- C is the point with position vector $(50\underline{i} + 136\underline{j})$
- D is the point with position vector $(22\underline{i} + 24\underline{j})$

(a) Show that AD is parallel to BC

(2 marks)

(continued on the next page)

6. continued.

Points A, B, C and D are used to model the vertices of a running track in the shape of a quadrilateral.

Runners complete one lap by running along all four sides of the track.

The lengths of the sides are measured in metres.

Given that a particular runner takes exactly 5 minutes to complete 2 laps,

(b) calculate the average speed of this runner, giving the answer in kilometres per hour.
(4 marks)

(Total for Question 6 is 6 marks)

7. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

A curve has equation

$$x^3 + 2xy + 3y^2 = 47$$

(a) Find $\frac{dy}{dx}$ in terms of x and y

(4 marks)

The point $P(-2, 5)$ lies on the curve.

(b) Find the equation of the normal to the curve at P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(3 marks)

(Total for Question 7 is 7 marks)

8. (a) Express $2\cos\theta + 8\sin\theta$ in the form $R\cos(\theta - \alpha)$, where R and α are constants,

$R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α in radians to 3 decimal places.

(3 marks)

(continued on the next page)

8. continued.

The first three terms of an arithmetic sequence are

$\cos x$

$\cos x + \sin x$

$\cos x + 2\sin x$

$x \neq n\pi$

Given that S_9 represents the sum of the first 9 terms of this sequence as x varies,

(b) (i) find the exact maximum value of S_9

(ii) deduce the smallest positive value of x at which this maximum value of S_9 occurs.

(3 marks)

(Total for Question 8 is 6 marks)

9. The curve C has parametric equations

$$x = t^2 + 6t - 16$$
$$y = 6\ln(t + 3)$$
$$t > -3$$

(a) Show that a Cartesian equation for C is

$$y = A\ln(x + B)$$
$$x > -B$$

where A and B are integers to be found.

(3 marks)

(continued on the next page)

9. continued.

The curve C cuts the y–axis at the point P

(b) Show that the equation of the tangent to C at P can be written in the form

$$ax + by = c\ln 5$$

where a, b and c are integers to be found.

(4 marks)

(Total for Question 9 is 7 marks)

10. $$f(x) = \frac{3kx - 18}{(x+4)(x-2)}$$

where k is a positive constant

(a) Express $f(x)$ in partial fractions in terms of k

(3 marks)

(b) Hence find the exact value of k for which

$$\int_{-3}^{1} f(x)\,dx = 21$$

(4 marks)

(Total for Question 10 is 7 marks)

11. Refer to Diagram 1, Diagram 2 and Diagram 3 for Question 11 in the Diagram Booklet.
You may be provided with a 3D model.

The diagrams and the 3D model show a tank in the shape of a cuboid being filled with water.
Diagram 1 and the 3D model show a 3D view.
Diagram 2 shows a front view.
Diagram 3 shows a side view.

The base of the tank measures 20 metres by 10 metres and the height of the tank is 5 metres, as shown in the diagrams.

At time t minutes after water started flowing into the tank the height of the water was h metres and the volume of water in the tank was $V\,m^3$

(continued on the next page)

11. continued.

In a model of this situation

- the sides of the tank have negligible thickness
- the rate of change of V is inversely proportional to the square root of h

(a) Show that

$$\frac{dh}{dt} = \frac{\lambda}{\sqrt{h}}$$

where λ is a constant.

(3 marks)

(continued on the next page)

11. continued.

Given that

- initially the height of the water in the tank was $1 \cdot 44$ metres

- exactly 8 minutes after water started flowing into the tank the height of the water was $3 \cdot 24$ metres

(b) use the model described on page 18 to find an equation linking h with t, giving your answer in the form

$$h^{\frac{3}{2}} = At + B$$

where A and B are constants to be found.

(5 marks)

(continued on the next page)

11. continued.

(c) Hence find the time taken, from when water started flowing into the tank, for the tank to be completely full.

(2 marks)

(Total for Question 11 is 10 marks)

12. Refer to the diagram for Question 12 in the Diagram Booklet.

The number of subscribers to two different music streaming companies is being monitored.

The number of subscribers, N_A, in thousands, to company A is modelled by the equation

$$N_A = |t - 3| + 4$$

$$t \geq 0$$

where t is the time in years since monitoring began.

The number of subscribers, N_B, in thousands, to company B is modelled by the equation

$$N_B = 8 - |2t - 6|$$

$$t \geq 0$$

where t is the time in years since monitoring began.

(continued on the next page)

12. continued.

The diagram shows a sketch of the graph of N_A and the graph of N_B over a 5–year period.

Use the equations of the models to answer parts (a), (b), (c) and (d).

(a) Find the initial difference between the number of subscribers to company A and the number of subscribers to company B

(2 marks)

(continued on the next page)

12. continued.

When $t = T$ company A reduced its subscription prices and the number of subscribers increased.

(b) Suggest a value for T, giving a reason for your answer.

(2 marks)

(c) Find the range of values of t for which $N_A > N_B$ giving your answer in set notation.

(5 marks)

(d) State a limitation of the model used for company B

(1 mark)

(Total for Question 12 is 10 marks)

13. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Find the first three terms, in ascending powers of x, of the binomial expansion of

$$(3 + x)^{-2}$$

writing each term in simplest form.
(4 marks)

(continued on the next page)

13. continued.

(b) Using the answer to part (a) and using algebraic integration, estimate the value of

$$\int_{0\cdot 2}^{0\cdot 4} \frac{6x}{(3+x)^2}\,dx$$

giving your answer to 4 significant figures.

(4 marks)

(continued on the next page)

13. continued.

(c) Find, using algebraic integration, the exact value of

$$\int_{0\cdot 2}^{0\cdot 4} \frac{6x}{(3+x)^2}\,dx$$

giving your answer in the form $a\ln b + c$, where a, b and c are constants to be found.
(5 marks)

(Total for Question 13 is 13 marks)

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14. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that the equation

$$2\tan\theta(8\cos\theta + 23\sin^2\theta) = 8\sin 2\theta(1 + \tan^2\theta)$$

may be written as

$$\sin 2\theta(A\cos^2\theta + B\cos\theta + C) = 0$$

where A, B and C are constants to be found.
(3 marks)

(continued on the next page)

14. continued.

(b) Hence, solve for $360° \leq x \leq 540°$

$$2\tan x(8\cos x + 23\sin^2 x) = 8\sin 2x(1 + \tan^2 x)$$

$x \in \mathbb{R}$

$x \neq 450°$

(4 marks)

(Total for Question 14 is 7 marks)

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15. A student attempts to answer the following question:

Given that x is an obtuse angle, use algebra to prove by contradiction that

$$\sin x - \cos x \geq 1$$

The student starts the proof with:

Assume that $\sin x - \cos x < 1$ when x is an obtuse angle

$$\Rightarrow (\sin x - \cos x)^2 < 1$$

$$\Rightarrow \ldots$$

(continued on the next page)

15. continued.

The start of the student's proof is reprinted below.

Complete the proof.

Assume that $\sin x - \cos x < 1$ when x is an obtuse angle

$$\Rightarrow (\sin x - \cos x)^2 < 1$$

(Total for Question 15 is 3 marks)

TOTAL FOR PAPER IS 100 MARKS

END OF PAPER
